Model Question Paper (Theory)
B.A/B.Sc. III Year Examination, March/April 2011
MATHEMATICS PAPER-III

Time:3Hrs **Maximum Marks:100**

NOTE: Answer 6 questions from Section- A and 4 questions from Section –B choosing atleast one from each unit. Each question in Section- A carries 6 marks and each question in Section-B carries 16 marks.

SECTION-A (6×6=36)

UNIT-I

1) Define a subspace. Prove that the intersection of two subspaces is again a subspace.

2) Define Linear transformation. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as

$T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3,\ a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ in to $V_2(R)$.

UNIT-II

3) Find all eigen values of the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

4) Define orthogonal set. Show that any orthogonal set of non-zero vectors in an inner product space V is linearly independent.

UNIT-III

5) Evaluate $\iint x^2y^2\, dxdy$ over the domain $\{ (x, y): x \geq 0; y \geq 0; (x^2 + y^2) \leq 1\}$.

6) Evaluate $\iint (x^2 + y^2)\, dxdy$ over the domain bounded by $xy = 1; y = 0; y = x; x = 2$.

UNIT-IV

7) Define irrotational vector. Show that $A = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is Irrotational. Find φ such that $A = \nabla\varphi$.

8) Evaluate $\iint A.n\, ds$ where A=18zi-12j+3yk and S is that part of the plane 2x+3y+6z=12 which is located in first octant.

SECTION-B (4×16=64)

UNIT-I

9) a) Define Basis of a vector space. Prove that any two basis of a finite dimensional vector Space $V(F)$ have same number of elements.

b) If W_1, W_2 are two subspaces of a finite dimensional vector Space $V(F)$ then $dim(W_1 + W_2) = dimW_1 + dimW_2 - \dim(W_1 \cap W_2)$.

10) a) State and prove Rank and Nullity theorem in linear transformation.

b) Show that linear operator T defined on R^3 by $T(x, y, z) = (x + z, x - z, y)$ is invertible. And hence find T^{-1}.

UNIT-II

11) a) Prove that distinct characteristic vectors of T corresponding to distinct characteristic of T are linearly independent.

b) Let T be the linear operator on R^3 which is represented in standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable.

12) a) State and prove schwarz's inequality .

b) Apply the Gram- Schmidt process to the vector $\beta_1 = (1,0,1)$; $\beta_2 = (1,0,-1)$; $\beta_3 = (0,3,4)$ to obtain an orthonormal basis for $V_3(R)$ with the standard inner product.

UNIT-III

13) a) Prove the sufficient condition for the existence of the integral.

b) Verify that $\iint_R (x^2 + y^2)dydx = \iint_R (x^2 + y^2)dxdy$ where the domain R is the triangle bounded by the lines $y = 0, y = x, x = 1$.

14)a) Prove the equivalence if a double integra with repeated integrals.

b) Evaluate the following integral: $\iint \frac{x-y}{x+y} dxdy$ over $[0,1;0,1]$.

UNIT-IV

15) a) For any vector A, Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla . \text{A}) - \nabla^2 A$.

b) If $U = 3x^2y; V = xz^2 - 2y$. Evaluate $grad\ [\ (gradU)\ .\ (gradV)\ ]$.

16) a) State and prove Green's theorem in a plane.

b) Verify stoke's theorem for $A = (2x - y)i - yz^2j - y^2zk$. where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.